

# 3D FDTD-LLG modelling of magnetisation dynamics in thin film ferromagnetic structures

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**Abstract.** Here I propose a model which uses 3D finite-difference-time-domain (FDTD) approach together with LLG to find the exact solutions for magnetisation dynamics in thin film ferromagnetic structures. As a benchmark testing I demonstrate application of such model for different classical phenomena such as Faraday effect, and then explore the dynamic characteristics of thin films in magnetostatic applications. In particular I consider propagation of magnetostatic/spin waves in metallic and metallised magneto-dielectric thin films and magnetic structures and demonstrate their dispersion characteristics.

**Introduction** There is a growing need in high frequency tunable microwave materials for applications in the areas of microwave electronics, transformation optics, photonics. Due to their intrinsic RF phenomena, such as FMR, ferromagnetic thin films have always been of great interest and led to a great amount of experimental research very often supported by numerical simulations. While purely magnetostatic solvers, such as OOMMF or Mumax, have always been the standard benchmark tools, if the properties of the material are non-uniform (e.g. metal/dielectric) one need to consider a full solution of Maxwell equations alongside the materialistic equations, such as e.g. Landau-Lifshits-Gilbert (LLG), providing the relation between the magnetisation and the magnetic field. Here we consider a model that uses 3D FDTD approach together with LLG to solve a number of typical magnetisation dynamics problems [1,2].

## 3D FDTD-LLG Formalism

Maxwell equations

$$\begin{cases} \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \\ \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \\ \frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H} \\ \vec{E}(\omega) = \frac{\vec{D}(\omega)}{\epsilon_0 \epsilon_r(\omega)} \end{cases}$$

Materialistic relations for  $\mu_r$  and  $\epsilon_r$

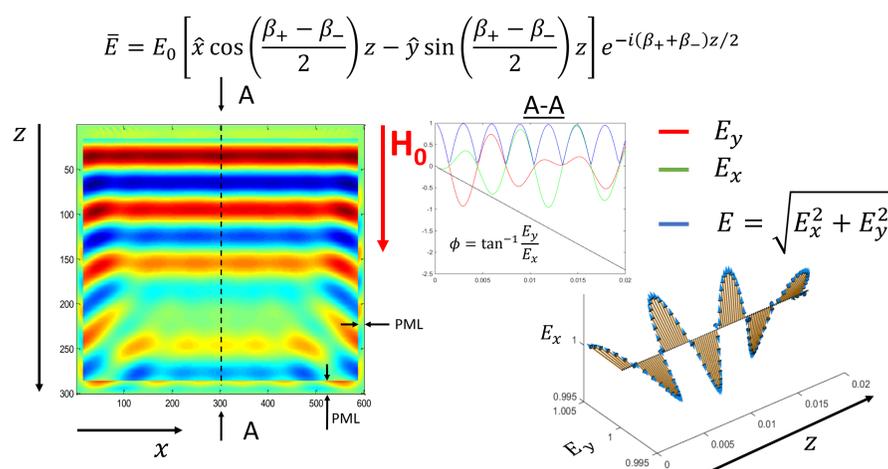
$$\frac{d\vec{M}}{dt} = -\gamma(\vec{M} \times \vec{H}_{eff}) + \frac{\alpha}{|\vec{M}|}(\vec{M} \times \frac{d\vec{M}}{dt})$$

$$\epsilon_r(\omega) = \epsilon_r + \frac{\sigma}{i\omega\epsilon_0}$$

where  $H, B, E, D$  are the magnetic and electric fields,  $M$  magnetisation and  $\mu_r$  and  $\epsilon_r$  are relative permeability and permittivity of the material,  $\sigma$  - conductivity,  $\omega$  - activation radial frequency.

## Benchmarking: Faraday rotation.

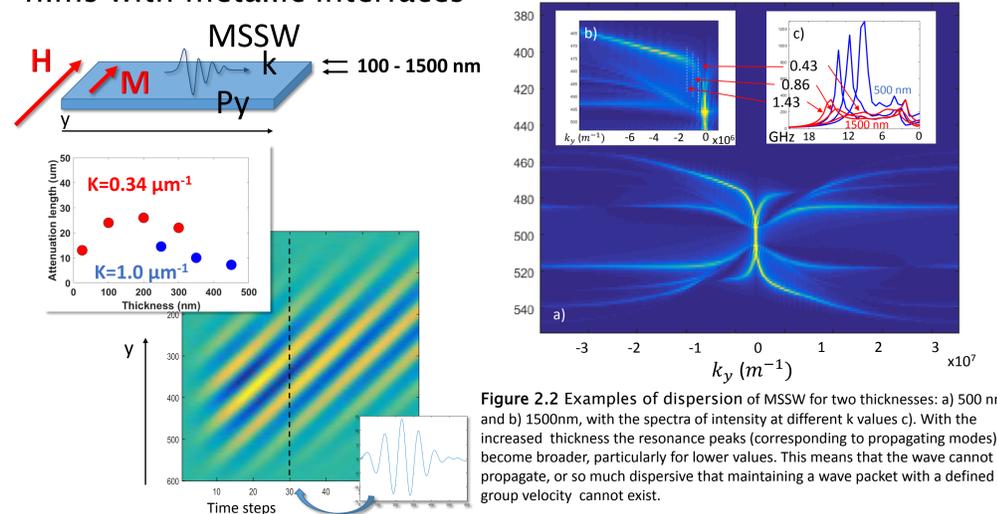
As a benchmark test the model was applied for calculation of several 'classical' examples where the solution can be found analytically. Here is the case of Faraday rotation of a plane wave propagating in a magneto-dielectric medium.



**Figure 1** Electric field inside the magneto-dielectric medium. a) Intensity map of  $E_y$  component inside the medium. b)  $E_y$ ,  $E_x$ ,  $E = \sqrt{E_x^2 + E_y^2}$  and  $\phi = \tan^{-1} \frac{E_y}{E_x} = -\left(\frac{\beta_+ - \beta_-}{2}\right)z$  along the A-A cut. c) Orientation of the electric field along the A-A cut. Polarisation vector rotates with constant phase  $\phi = 0.45$  (analytical value 0.46).  $4\pi M_s = 1.8$  kOe,  $H = 6$  kOe,  $f = 11$  GHz,  $\epsilon_r = 14$ . The chosen frequency is sufficiently off the resonance ( $\gamma H/2\pi \approx 20$  GHz), the ellipticity is negligible and both RHCP and LHCP modes present. The example also shows the imperfectness of PML layers. Because of the reflections on the sides the wave fronts become distorted. To avoid this the periodic boundary conditions can be applied (e.g. as in the example with magnetostatic waves above).

## Magnetostatic waves in thin metallic films.

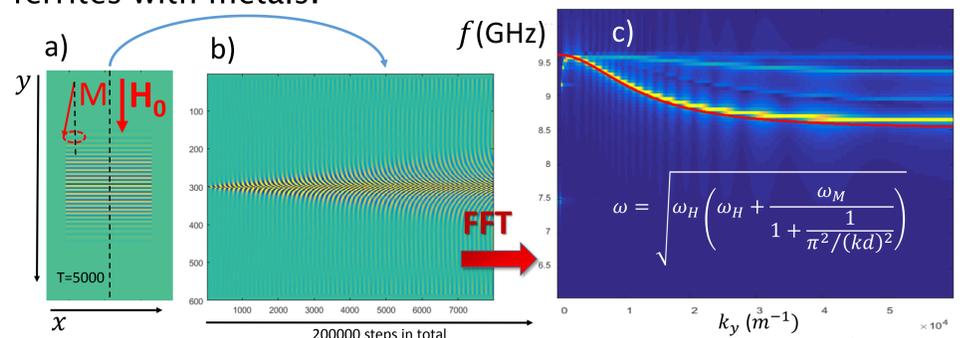
The model can be applied for calculation of magnetostatic waves and their dispersions. A particular advantage of this approach is in the cases in which magnetic thin films are metals or being used together with metallic layers/interfaces. Here I consider three examples for calculation of different type of magnetostatic waves in metals and magnetodielectric films with metallic interfaces



**Figure 2.1** Magnetostatic Surface Wave (MSSW) configuration a) and the continuous wave excitation using FDTD-LLG b). The waves are considered in permalloy films of 100 to 1500 nm. The excitation is in Damon-Eshbach mode using continuous wave packet with a predetermined frequency and wave number. The lower inset shows the propagating wave packet at a certain time-step. As the packet propagates the amplitude decreases due to the intrinsic damping ( $\alpha = 0.01$ ), but also due to the electric conduction. With larger thickness, the second factor dominates leading to decrease of the attenuation length. The upper inset shows how the attenuation length vs thickness in experimental measurements [5] and calculations with FDTD-LLG.

## Metalized ferrites.

Demonstrates non-reciprocity effect and dispersions in ferrites with metals.



**Figure 3** Backward volume modes in a metalized ferrite slab. a) Intensity map of  $M_x$  component in the middle of the slab in the time frame  $T=5000$ . Magnetisation is saturated along  $y$  coordinate. In each time frame, a 2D cut is made across the propagation direction and collected in a matrix of all times b). After 2D Fourier transform, the dispersion relation  $\omega(k)$  is obtained c). The red line indicates an analytical solution based on the equation of the first order backward volume mode [4]. In the model I used cell size of 50 microns, with 5 layers of magnetodielectric. Periodic boundary conditions applied in  $x$  and  $y$  directions. Excitation is obtained with a Gaussian pulse ( $\approx 5$  ps) in the middle pixel of the slab.

## Surface modes in a slab with one metalized surface.

The dispersion characteristic of the wave propagating in the slab with metal plate on its top b) The metal plate is on the bottom of the slab. The slab is magnetised tangentially (along  $x$  direction). The wave dispersion is non-reciprocal and depends on the propagation direction, orientation of the field/magnetisation and the position of the metal surface (top/bottom). Inset shows the dispersion relation and the magnetisation/field intensity through the thickness of the slab at 11 GHz for the case of metal on the top.

## References

- [1] M. M. Aziz, Progress In Electromagnetics Research B 15, (2009) 1–29 [2] M. Kostylev et al. J. Appl. Phys. 113 (2013) 043927 [3] D.M. Pozar, 'Microwave Engineering', John Wiley & Sons, © 2012 [4] A.G. Gurevich & G.A. Melkov, 'Oscillations and Waves', CRC Press, © 1996 [5] T. Manago et al. JJAP. 54 (2015) 113001.